

QUANTUM 215B

HW 1

Harmonic Oscillator Electron in an Electric Field

1. A one-dimensional harmonic oscillator of charge e is perturbed by an electric field of strength E in the positive x -direction i.e. $\vec{E} = E\hat{x}$. Calculate the change in each energy level to second order in the perturbation. Show that this problem can be solved exactly, and compare the result with the perturbation approximation.

Anharmonic Oscillator

2. Consider an anharmonic oscillator with a small quartic perturbing nonlinearity, i.e. $H_1 = \lambda x^4$.
 - (a) Use time-independent perturbation theory to show that the lowest (nontrivial) order expression for the n -th excited state is given by $E_n = \hbar\omega(n + 1/2) + \frac{3\hbar^2\lambda}{4m^2\omega^2}[1 + 2n + 2n^2]$
 - (b) Argue that no matter how small λ is, the perturbation expansion will break down for some large enough n . What is the physical reason?

Hint: It could be useful to use raising and lowering operators, rather than work in the coordinate representation (i.e. the differential equation).

Spin-1 Particle

3. A spin-1 particle characterized by the Hamiltonian $H = AS_z^2 + B(S_x^2 - S_y^2)$ with $B \ll A$.
 - (a) Compute the spectrum and the eigenstates to 0th order in B . Sketch the spectrum.
 - (b) Treating the B term as a perturbation compute its effects on all the eigenstates (spinors) and the spectrum to 1st order in B .

Sketch the spectrum, showing how it changes from $A=B=0$, to $A \neq 0, B=0$, and finally $A \neq 0, B \neq 0$, labeling all the states, and their corresponding energies.

Sakurai Problems

Chapter 5: 4, 6, 7, 11

Please do not wait until the last minute to do this HW set

Date: Due 1/15/10.